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DISCRETENESS, CONTINUITY AND HIERARCHICAL SCALING IN THE ARTS

Abstract

The interaction of the continuous and the discrete in aesthetic perception and creativity is shown to lead to the formation of a hierarchy of zone structures akin to musical scales. The hierarchical relations between operations, actions and activities underlying the formal features of the scaling processes in the arts are treated from the viewpoint of the general theory of activity. The formal synthesis of discreteness and continuity in a work of art is related to the meaning and sense of aesthetic message passed in conscious and unconscious communication. The fusion of the continuous and discrete sides of communication through the psychological mechanism of a set is discussed in brief.

1. Introduction

The problem of continuity and discreteness in the arts has a long history. Recently, it has acquired a novel aspect in the light of agitated Internet discussions on the nature of consciousness and the possibility of its modelling in computers [1]. From the viewpoint of the orthodox computationalism, everything in human psychology can be reduced to simple acts of binary discrimination, albeit in a most intricate combinations. Traditional binary logic should hence be quite enough to describe consciousness, and implementing its fundamental features in computer programs might be merely a matter of time. The adherents of the opposite view stress the apparent continuity of human feelings and activities to advocate the principal irreducibility of consciousness to discreet logic and computation; Goedel's theorems are commonly mentioned as an indication of that the traditional logic is intrinsically insufficient to be applied to subjectivity proper. The problem becomes further complicated with the observation that computation can be analogue as well as discrete, and the methods of discrete mathematics may well applicable to continuous systems, and inversely, apparently discrete behaviour may be a manifestation of continuous dynamics.

As it often happens with such controversies, the solution has to be sought for in a kind of synthesis, so that both discreteness and continuity would occupy their own place in the integrity of the whole. In this paper, I will try to outline an approach to the problem of discreteness and continuity in the arts, assuming that both aesthetic perception and artistic creativity obey common laws of hierarchical scaling, combining the distinction of well isolated entities and the infinite variability of modes of their actualisation. The mathematical model illustrating this approach has been described in detail earlier [2–4], and it will be used to illustrate the formal aspects of the problem in Section 2. Psychological considerations will be employed in Section 3 to discuss the qualitative aspect of human communication and the forms of discreteness and continuity in it. The concluding remarks (Section 4) will concern the ways of synthesising the continuous and the discrete in the same aesthetic phenomenon.

2. Information theory and the hierarchy of scales

The notions of information theory have already been applied by several authors to quantum physics [5, 6] within its own range of problems. In the model by Avdeev and Ivanov [2], quantum ideas and

information theory have been combined in a less straightforward way to provide a description of a quite different sphere, that of pitch perception. The possible sensations of sound frequency form a continuous (one-dimensional) manifold, which can be modelled by the Hilbert space of the possible states of a quantum system parametrised by the real number $h = \log f$. However, the actual perception of a tone can never be represented by a pure state h, but rather by a mixed state (wave packet) including many sound frequencies with different weights. Every such mixed state can be associated with a density operator G acting on the original pure-state space, so that the probability density to find the system in the pure state h will be given by the diagonal matrix element $G(h)$. Thus the continuum of pure states gets broken into a number of wave packets that can interact in perception as self-contained entities; this may be compared to collective effects in physics (solitons, phonons, laser modes, autoionising states etc.).

Extending the usual techniques of quantum physics to the density-operator level, one could assume that the interaction of the mixed states G_1 and G_2 would produce new operator states

$$\begin{aligned} G_1' &= (1 + \mu_{11})G_1 + \mu_{12}G_2 \\ G_2' &= \mu_{21}G_1 + (1 + \mu_{22})G_2 \end{aligned} \tag{1}$$

Here is the place where information theory enters the game. The cross-entropy

$$H_c = \int \rho'(x) \ln\left(\frac{\rho'(x)}{\rho(x)}\right) \tag{2}$$

can be used to measure the change in the original distributions due to interaction. After having accepted that the form of the distribution $G(h)$ remains the same for all the elementary stimuli of the same level, and choosing the Gauss distribution with the same dispersion σ for the standard wave packet, the Golitsyn's formula [7] for the quantity if information introduced by the interaction of two wave packets can be easily obtained [2]:

$$d(R) \sim \frac{R^2}{\sigma^2} \exp\left(-\frac{R^2}{\sigma^2}\right), \tag{3}$$

where $R = h_1 - h_2$ is the interval between the two elementary stimuli, h_1 and h_2 being the centres of the respective Gauss distributions. The value (3) can be considered as an average $\langle G_1^* G_2 \rangle$ of the product of the operators G_1 and G_2 representing the two interacting mixed states; it has been named *discordance* in [2], being the measure of mutual interference of two stimuli.

Formula (3) can be used as a basis for a wide range of aesthetic speculations. For our purpose it is important to note that it can be treated as expressing the unity of continuity and discreteness on the lowest level, in the operation of comparing two simple mental formations. The elementary stimuli are essentially discrete, but they represent a continuum of possible states, organising them into different hierarchical structures. The rearrangement of the components of the same hierarchy producing a different order is known as *refolding* in the hierarchical approach [8]. There are no "pure" ideas, characterised by a single point in the H space; however, any single element of the continuum may become the topmost element of a hierarchical structure, with all the rest of the space retained on the lower levels.

The introduction of the standard distribution $G_\sigma(h-h_0)$ parametrised by the position of its centre h_0 is a kind of the "negation of negation", restoring the continuum of possible states on the density-operator level. This continuity can be further quantised considering realistic stimuli as *discrete structures*. Thus, musical tones can be represented by the linear combinations $G = \sum_{k=1}^{N} t_k G_\sigma(h - h_k)$, with $h_k = h_1 + \log k$, which correspond to the internal (subjective) idea of a musical tone rather than to

the physical spectrum; hence, the collections of amplitudes $\{t_k\}$ ("internal timbres", [2]) should be set the same for all the musical tones, thus representing *perceptive tuning*. The discordance of two such compound tones is calculated as

$$\Delta(h_1^{(a)} - h_1^{(b)}) = \sum_{k,m=1}^{N} t_k t_m \left\langle G^{*(a)} G^{(b)} \right\rangle = \sum_{k,m=1}^{N} t_k t_m d(h_1^{(a)} - h_1^{(b)} + \log(k/m)), \qquad (4)$$

with $d(h)$ defined by (3) and the indices (a) and (b) referring to the two structures to compare. Once again, we have the unity of the continuous and the discrete on this level, the discrete structures being characterised by a continuous parameter, the pitch of the principal tone. Such discrete structures are similar to coherent mixtures in quantum mechanics.
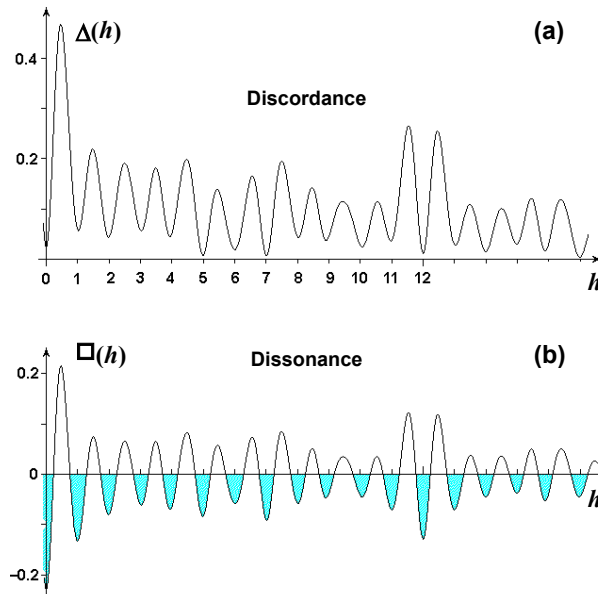

Figure 1. Discordance and dissonance functions for the 12-zone scale.

Being general enough, the discordance function (4) says nothing about the specifically aesthetic side of perception. However, the positions of its minima (the point of the lowest discordance) for some of timbres form a kind of integrity quite resembling musical modes (Fig. 1a). However, the correct definition of a scale must involve higher-level ideas, rather than mere picking out the points of minima. In [2], the *dissonance function* has been introduced as the difference of the discordance function $\Delta(h)$ and a locally determined medium level (assuming universal perceptive accommodation):

$$\square(h) = \Delta(h) - \int \frac{1}{\sigma\sqrt{2\pi}} \exp\left(-\frac{(x-h)^2}{2\sigma^2}\right) \Delta(x) dx \qquad (5)$$

The intervals with negative dissonance determine a *zone structure*, which is identified with an aesthetic scale (Fig. 1b). In other words, the aesthetic category "dissonance" now means that a structure perceived does not belong to any one of a pre-set collection of zones, so that speaking about consonance and dissonance may have sense only within a certain scale context. Nothing can be said about any isolated pitch or pitch interval, and the intervals dissonant in one scale may be consonant in another.

A zone scale is one of the most manifest instances of the synthesis of discreteness and continuity, being intimately related to both the organisation of the phenomena perceived and the ways of categorisation. However, there is also a higher level, that of cultural acceptance of a subjective structure. One has to account for that different scales have different chances to become a cultural phenomenon rather than an individual fantasy. The considerations of robustness and regularity suggest

that only a small number scales can acquire universal significance, the traditionally known musical systems being among them [2]. The corresponding "internal timbres" are called "optimal" in the hierarchical model described. Such structures may have fundamental significance for general aesthetics, since they definitely correlate with certain stages of cultural development, embracing all the arts, as well as the other levels of culture [4, 9].

Though a scale may be built starting from any key tone, reproducing the same zone structure at a different pitch, the combinations of scales in the same piece of music cannot be arbitrary, being restricted by a hierarchy of sub-scales embedded into the main (lowest-level) scale. This hierarchy naturally appears in the model [2], so that the collection of different-level scales with different central tones may become a rigorous definition of a pitch context. Musical modes and chords within the same well-tempered scale can serve as an example. Thus the possibility of the continuous adjustment of intonation within a pitch zone gets enhanced by the continuum of possible attributions of an interval on a number of distinct pitch scale levels. These multiple attributions constitute the basis of the aesthetic usage of pitch in music.

3. Discreteness and continuity in the theory of activity

To discover the universal significance of hierarchical scaling for the arts, one has to recall the basic facts from the general psychological theory of activity [10]. The three basic levels in the hierarchy of human behaviour are *operation*, *action* and *activity*; some relations between them are schematically shown in Fig. 2. An activity (driven by some motive and following some purpose) is always being actualised as a sequence of actions, and there may be different sequences of actions actualising the same activity; in other words, an activity is a hierarchy of actions. Every action (with its specific reason and goal) can be further implemented as a sequence of operations; this can also be achieved in different ways, so that an action can as well be treated as a hierarchy of operations. An operation is the most elementary (most folded) kind of act; subjectively, it is performed in no time, and corresponds to the abstraction of a point. Hence, operations represent the discrete side of human behaviour; on the opposite, activity represents the continuity, being essentially a process with a definite direction but no marked beginning or end. An action is the unity of continuity and discreteness, being limited in time: one can complete an action and obtain a result—still, it lasts, never being as syncretic as operations; the idea of a segment of a real axes comes to mind.

To become integrity, human behaviour must contain all the three levels. It is in their interaction that the phenomenon of consciousness can only be formed. Basically, consciousness is a characteristic of the level of action, while operations and activities can be associated with the two opposite kinds of the unconscious, the subconscious and the superconscious, respectively. In this picture, consciousness may be said to be a *boundary* of a personality, while its interior is unconscious and its immediate social environment ("the zone of imminent development", [11]) influences behaviour as a kind of super-consciousness.

As a kind of communication, art is just another instance of activity; consequently, it cannot avoid being the unity of the continuous and the discrete, as long as it remains communication, implying information exchange between the conscious beings. One can easily see that the discrete side of an aesthetic "message" will be related to operation level, while its continuity will link it to some activity. The content of the "message" (information proper) will always be a kind of interval, a continuous process within discrete limits.

Since any hierarchy can be folded an unfolded in different ways, activities may fold into actions, and actions fold into operations—inversely, operations and actions may unfold into actions and activities respectively (Fig. 2). Thus, an activity becomes an action when it becomes included as a

part to another activity, thus losing its self-motion. On the other hand, an action may become self-contained enough to evolve into an activity, when the former goals do not matter ever more. Also, a frequently repeated action may achieve a level of automation which would not require any conscious goals, thus becoming subconsciously performed operations. Inversely, when a person's attention turns to the way of performing an operation (as in the case of an obstacle encountered), the operation becomes an action, which may sometimes cause a change of activity, with the corresponding restructuring of motivation. Unfolding operations into actions and even into activities is a fundamental method of psychological experiment, when the deep mechanisms of behaviour are being explored. For instance, a psychophysical experiment with a person determining the subjective pitch of a tone may explicate the Gauss distribution taken for the form of a harmonic in the informational model described in the previous section. Moreover, if one discovers an analogous activity when studying a different sensory modality, it can be quite logically deduced that this activity may fold into operations and actions similar to those already known in pitch perception, and the same mathematical model can be used to describe a different class of aesthetic phenomena. Thus directional scales in the visual arts have been introduced in [4], and universal hierarchical scaling has been considered as an important cultural mechanism related to the periodicity in cultural development characterised by the idea of culture-historical formations [4, 9, 12].
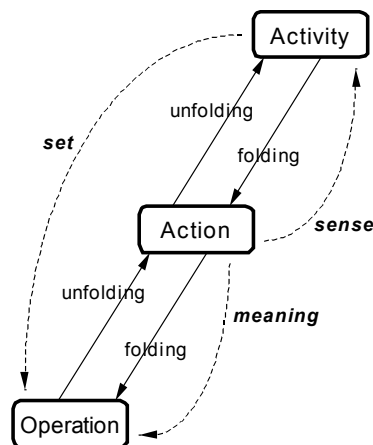

Figure 2. The hierarchy of activity.

Considering art as a form of communication in the framework of the general theory of activity helps to clarify the two aspects that make a work of art "informative" enough to acquire a universal significance characteristic of spirituality, which is an indispensable feature of true art: an aesthetic "message" must be *meaningful* and have *sense*. This point is critical in deciding whether something can be called a work of art or not, especially in abstract art. The thing may be not beautiful and even ugly—it may have nothing sublime in it—still that will be art as long as it can cause resonance in the people thus accentuating their universality and unity. The theory of activity gives clue to the explanation of this. As indicated in Fig. 2, the *meaning* of an action is just the hierarchy of operations that can become its means, while its *sense* is nothing but the position in an embracing activity. The numerous psychological consequences of such a distinction are beyond the scope of this article—but the necessity of both sense and meaning in a work of art is now evident from the demand of behavioural integrity formulated above.

To be of any cultural significance, a work of art must be based on universal operations, available to every member of the society. The existence of such operations is the basis of the "interpretability" of aesthetic message, projecting it into the individual experience of every person. For

those who do not share the same operational background the message will remain meaningless until the cultural schemes implied get somehow learned. Thus, aesthetic meaningfulness can be achieved through using a definite hierarchy of scales, providing the basis for common interpretations within a specific culture. It is clear that too much experimenting with forms in the arts may lead to meaningless things that nobody (including the author) recognises as the works of art; to establish cultural links, a work of art must employ traditional elements, albeit unexpectedly organised. The new has to grow from within the old. Luckily, few people can invent anything absolutely uncommon, and the major problems with the meaning of art arise from cultural incompatibility of different societies or social layers.

However, mere presence of meaning cannot guarantee aesthetic quality. To efficiently communicate, different people must be involved in at least one common activity, so that the message produced by one person could have sense for the others. Otherwise, the work is bound to be perceived as "empty", or "dull", never possessing the motivating power of true art. Things pretending to be spiritual (in particular, artistic) must carry an idea in them; no random play of imagination or productive forces can ever result in a work of art. It does not mean that there is no place for randomness in the arts—sometimes, it may well be used to achieve certain aesthetic effect; still, that would always imply quite definite aesthetic context, "synchronising" the activities of the people producing art and consuming it. For instance, alleatoric music may seem devoid any sense to the people unless they have ever experienced doing something like that in their life; an idea penetrates such music through the peculiar way of selecting quite definite sequences of notes for the musical piece, preferring them to many other sequences which do not differ from the chosen ones save in the eyes of the composer engaged in certain activity. In the informational model of [2–4], the sense may be conveyed by the special organisation of pitch context, assuming pre-designed relations in the hierarchy of embedded scales. As soon as the listener can share the same scale hierarchies with the composer, the musical piece will be comprehensible to them.

The aspects of continuity and discreteness related to meaning and sense are not specific for art, since they characterise any communication as a kind of activity. However, the discussion of the idea of the so called non-communication art on the Internet [14] shows that there is almost unanimous consent that communication lies in the core of art, and hence its basic features must influence any kind of aesthetic perception.

4. Continuous discreteness and discrete continuity

The two approaches to the problem of the continuous and the discrete in the arts described above are the two sides of the same aesthetic position, according to which it is the unity of different properties and kinds of behaviour that can lead to the kind of integrity implied by the idea of art. However, the unity of the opposites can only be achieved through their mutual penetration (reflection) and interchangeability of their positions within the whole. The hints to the mechanisms of such mutability can be found in the processes of folding and unfolding of hierarchies, and the hierarchy of human behaviour in particular. Thus, most elaborate structures may fold into points when viewed from the side of their relations to the other structures of the same level. Mathematically, this could be modelled by assigning a continuum-valued parameter to the whole structure and then comparing the structures by the values of that parameter. In the behavioural approach, this folding of structures into points corresponds to unfolding the action enumerating the components of a discrete structure and their links into an activity similar to continuous measurement. The opposite formal procedure is unfolding a point in the continuum into a discrete structure; the typical example is the overtones of musical tone. Such an unfolding usually involves an iterative process considered simultaneously, which corresponds to

folding repeated actions into operations, as described in the previous section. The opposite direction of the folding/unfolding processes in the phenomenological and psychological hierarchies is an important feature that can be used to determine the correct direction of historical development in any particular case.

The formal description of aesthetic phenomena often omits any references to conscious action, the level of behaviour linking operations to activities. The direct influence of activity on operations is reflected in the psychological notion of a *set*. Thus, the effect of any operation can only be expressed in certain forms, and the collection of these forms is pre-defined by the current activity. For example, people's sensations and feelings are always clipped by their current perceptual mood, which may cause numerous kinds of illusions and misconceptions. In the model of pitch perception described in [3], sets were represented by the elementary conceptions (Gauss distributions), the internal timbres (a collection of formants), or the pitch contexts (the hierarchies of embedded scales), depending on the level of consideration. The refoldability of hierarchies may lead to a change in the relations between their levels, so that the non-direct links like sets may become quite real processes, with their own physical mechanisms. The fusion of discreteness and continuity thus obtained leads to an important aesthetic phenomenon which was called an "aesthetic interval" in [14]. It is well-known that, to be interesting, a thing must be new, but not too novel. On the other side, there must be a balance between the realistic imaging and free expressiveness, so that a work of art could be identified with *some* reality still remaining clearly recognised as artificial and abstract enough. The relations like that are well described by the functions of the discordance type (3), which is no wonder, since the merging of the opposite levels of the behavioural hierarchy is normal for any formal (and especially mathematical) scheme. In reflection, this may leads to the idea of absolute *a priori* rules, given to the people from outside. Thus, there is a popular opinion that aesthetic properties can be possessed by natural things and phenomena, and that people can only discover these natural features (beauty, perfection etc.). Quite logically, this provokes the attempts to find the objective correlates of aesthetic phenomena: musical consonance and dissonance become attributed to the sound frequency ratios, visual harmony gets associated with some proportions, and so on. However, this superficial objectivity is as erroneous as the mystic attribution of any aesthetic feeling or creative gift to an abstract spirit, gods, or a play of subjectivity in itself. Sets *do* come to the people from the outside—but this "outside" is not the physical reality or a purely ideal substance, but rather people's communication uniting them into society. Considering the processes of hierarchical refolding can explicate the otherwise hidden relations and thus provide a solid basis for informational description of art.

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